

Abstraction-based Incremental Inductive Coverability for Petri nets

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Abstraction

- Check the coverability problem of Petri nets
- Combine IC3 with place-merge abstraction (IC3+PMA)

Outline

- ① Preliminaries
- ② IC3 algorithm for PN
- ③ Place-merge abstraction (PMA)
- ④ IC3+PMA algorithm
- ⑤ Experiments

Definition

A Petri net is a tuple $N = \langle P, T, W, m_0 \rangle$ where:

- P is a finite set of places
- T is a finite set of transitions such that $P \cap T = \emptyset$
- W is an arc function: $(P \times T) \cup (T \times P) \rightarrow \mathbb{N}$ describing the relationship between places and transitions
- m_0 is the initial marking. A marking $m \in \mathbb{N}^{|P|}$ is a vector specifying a number $m(p)$ of tokens for each place $p \in P$.

for vector $m_1, m_2 \in \mathbb{N}^{|P|}$
 $m_1 \preceq m_2$ iff for every $p \in P: m_1(p) \leq m_2(p)$

Definition

Let N be a Petri net.

- $pre(m) = \{m' \mid \exists t \in T: m' \rightarrow m\}$

- $Reach_i$ contains all reachable markings from m_0 within i steps.

- $Reach = \bigcup_{i \geq 0} Reach_i$ contains all reachable markings from m_0 .

Coverability problem

Let N be a Petri net, m_t the target marking.

- The coverability problem is to prove whether there exists a reachable marking $m_r \in Reach$ such that $m_t \preceq m_r$.

Coverability problem

Let N be a Petri net, m_t the target marking.

- The coverability problem is to prove whether there exists a reachable marking $m_r \in Reach$ such that $m_t \preceq m_r$.
- The coverable set of N within i steps is $Cover_i = Reach_i^\downarrow$
- The coverable set of N is $Cover = Reach^\downarrow$

IC3 algorithm for Petri nets

IC3 is a state-of-art of model checking

Efficient implementation of IC3 to check the coverability problem of Petri nets without using SMT solvers

IC3 algorithm for Petri nets

IC3 maintains a sequence $F_0, F_1 \dots F_k$

where F_i is a downward-closed set called *frame* that over-approximates the coverable set within i steps.

The algorithm generally proceeds by alternating two phases: the blocking phase and the propagation phase.

IC3 algorithm for Petri nets

Blocking phase: $block(a, i)$

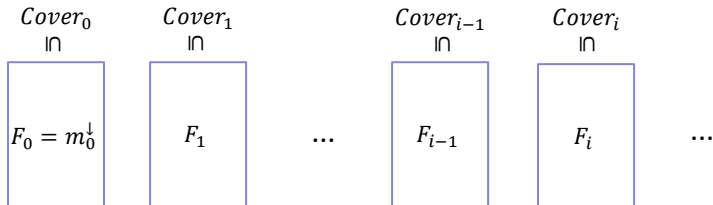
IC3 algorithm for Petri nets

Blocking phase: $block(a, i)$ \longrightarrow

try to prove a^\uparrow is
unreachable within i steps

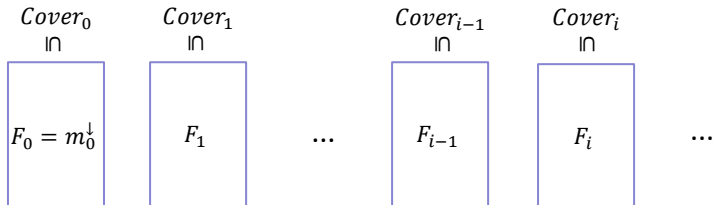
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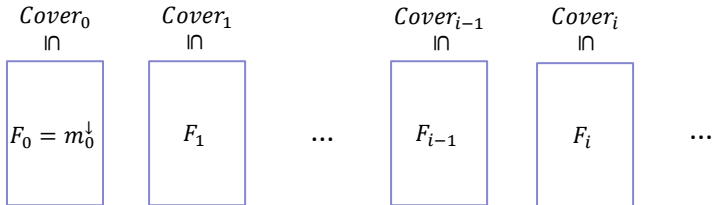
Blocking phase: $block(a, i)$



given a pair (a, i)

IC3 algorithm for Petri nets

Blocking phase: $block(a, i)$

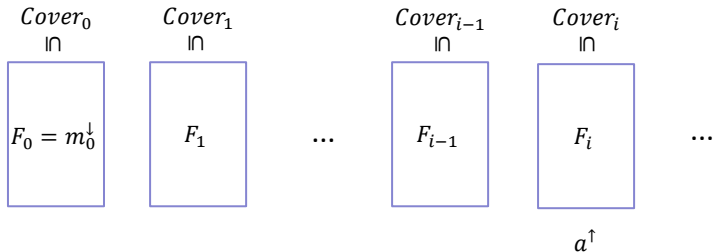


given a pair (a, i)

**try to prove a^\uparrow is
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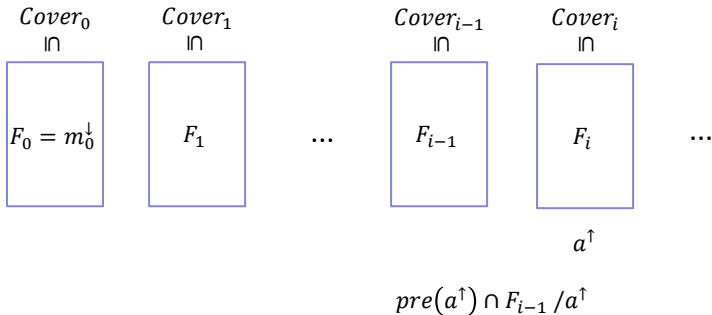
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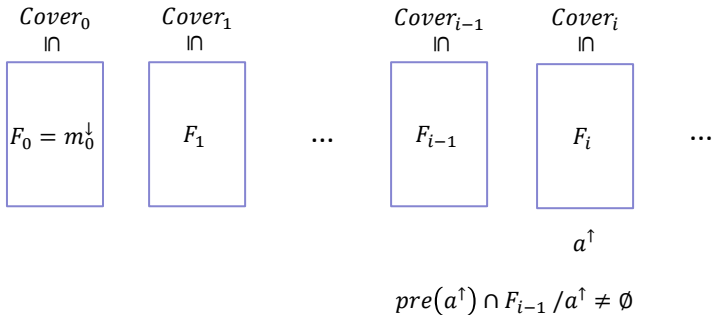
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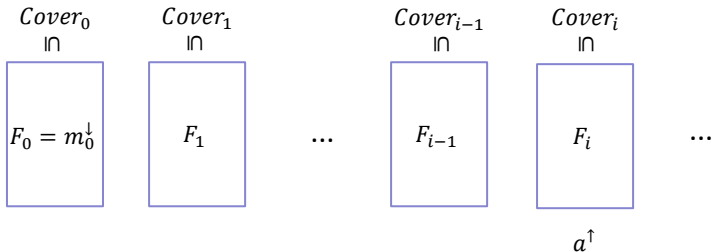
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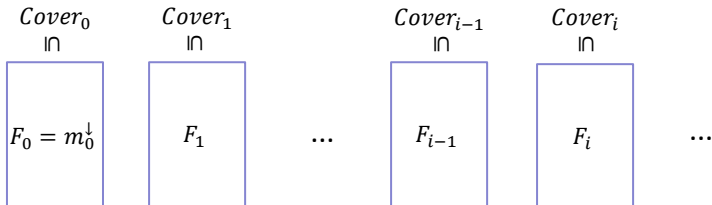


$$pre(a^\uparrow) \cap F_{i-1} / a^\uparrow \neq \emptyset$$

extract an unselected marking b
from $pre(a^\uparrow) \cap F_{i-1} / a^\uparrow$

IC3 algorithm for Petri nets

Blocking phase: $block(a, i)$



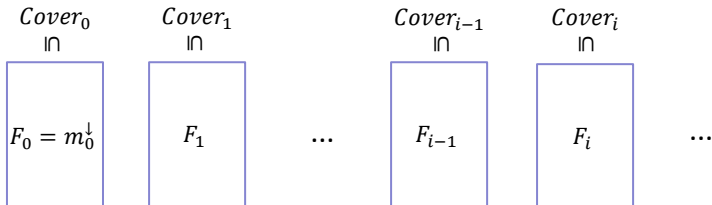
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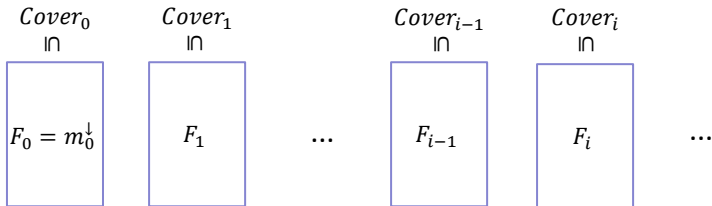
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IC3 algorithm for Petri nets

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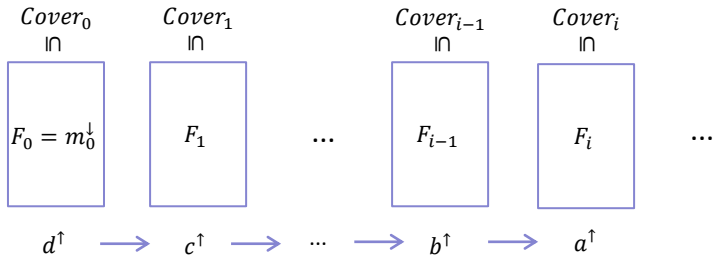
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generate a new pair $(b, i - 1)$
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try to prove b^\uparrow is unreachable
within $i - 1$ steps

IC3 algorithm for Petri nets

Blocking phase: $block(a, i)$



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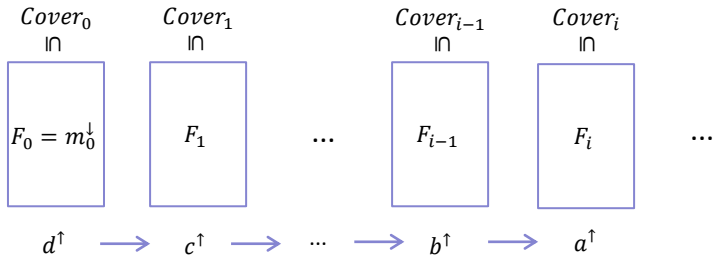
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IC3 algorithm for Petri nets

Blocking phase: $block(a, i)$



finally generate a new pair $(d, 0)$

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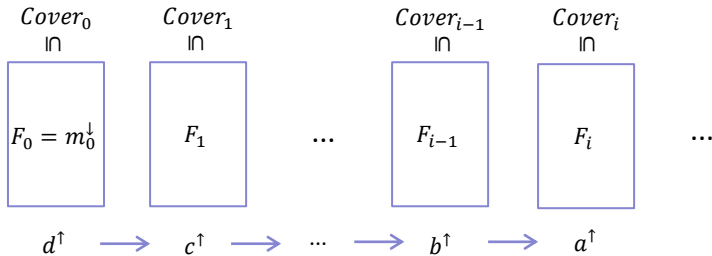
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IC3 algorithm for Petri nets

Blocking phase: $block(a, i)$



finally generate a new pair $(d, 0)$

find a path from m_0^\downarrow to a^\uparrow

$$pre(a^\uparrow) \cap F_{i-1} / a^\uparrow \neq \emptyset$$

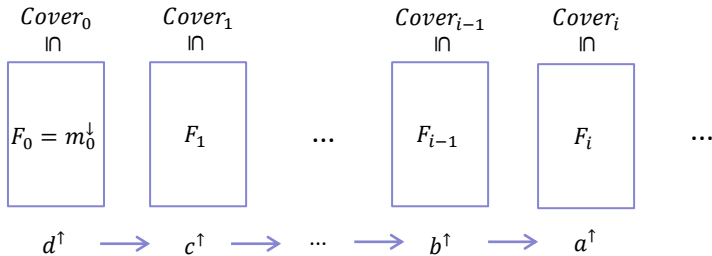
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IC3 algorithm for Petri nets

Blocking phase: $block(a, i)$



finally generate a new pair $(d, 0)$

find a path from m_0^\downarrow to a^\uparrow

fail to block a at F_i

i.e. a is coverable

$pre(a^\uparrow) \cap F_{i-1} / a^\uparrow \neq \emptyset$

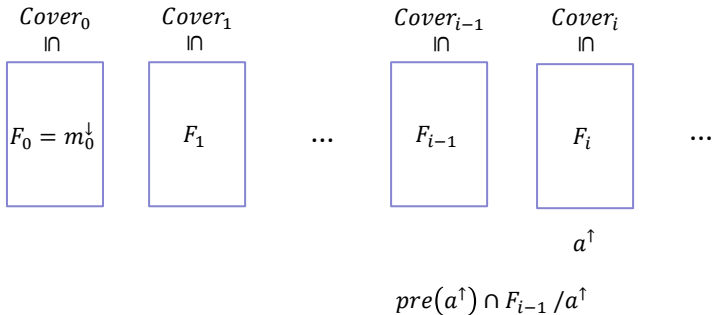
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generate a new pair $(b, i - 1)$
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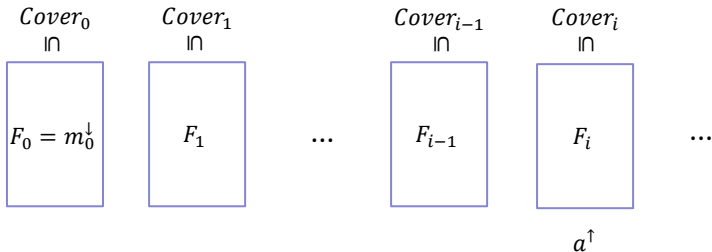
IC3 algorithm for Petri nets

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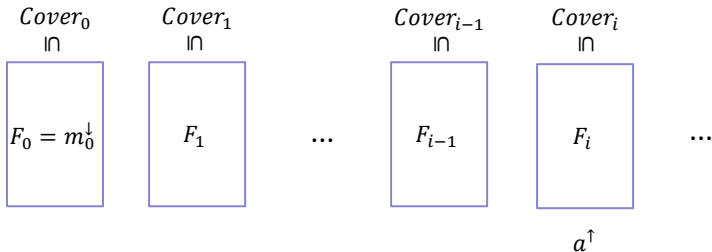
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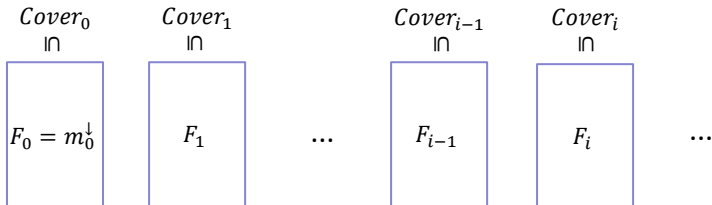


$$pre(a^\uparrow) \cap F_{i-1} / a^\uparrow = \emptyset$$

a^\uparrow cannot be reachable in 1 step
from $Cover_{i-1}$

IC3 algorithm for Petri nets

Blocking phase: $block(a, i)$



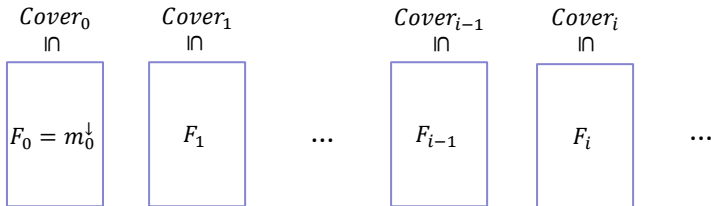
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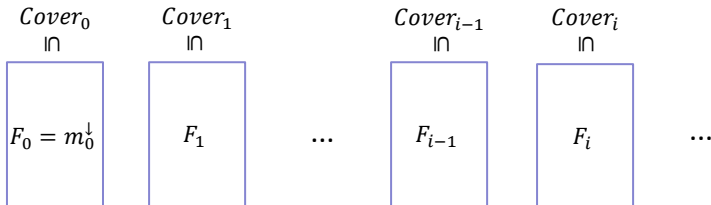
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a is uncovered within i steps

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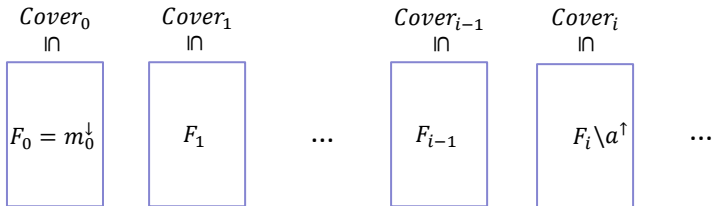
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a^\uparrow can be removed from the
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IC3 algorithm for Petri nets

input $N = \langle P, T, W, m_0 \rangle$ and m_t
initialize $F_0 = m_0^\downarrow, F_1 = \mathbb{N}^{|P|}, k = 1$

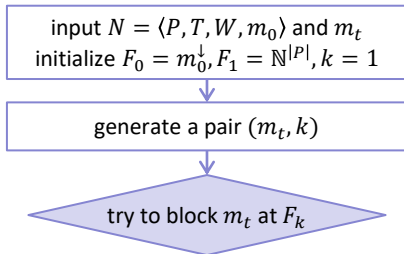
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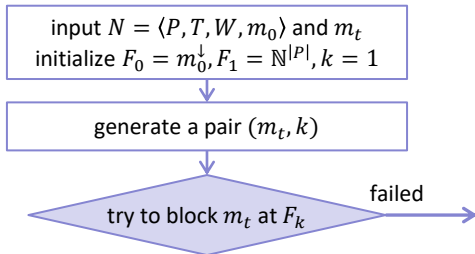


generate a pair (m_t, k)

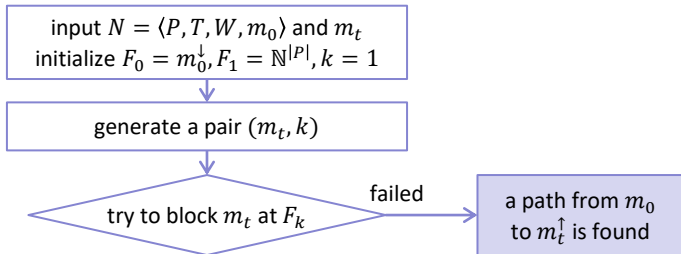
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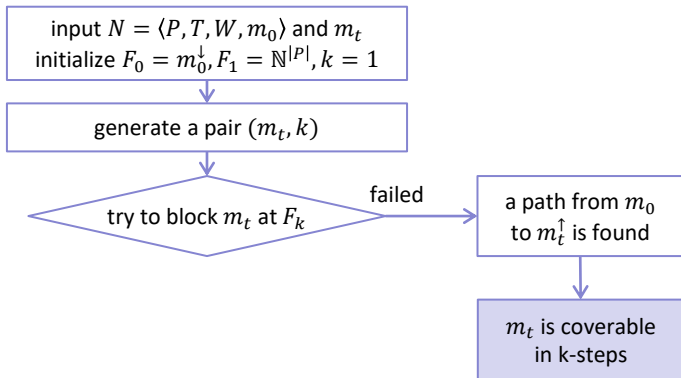
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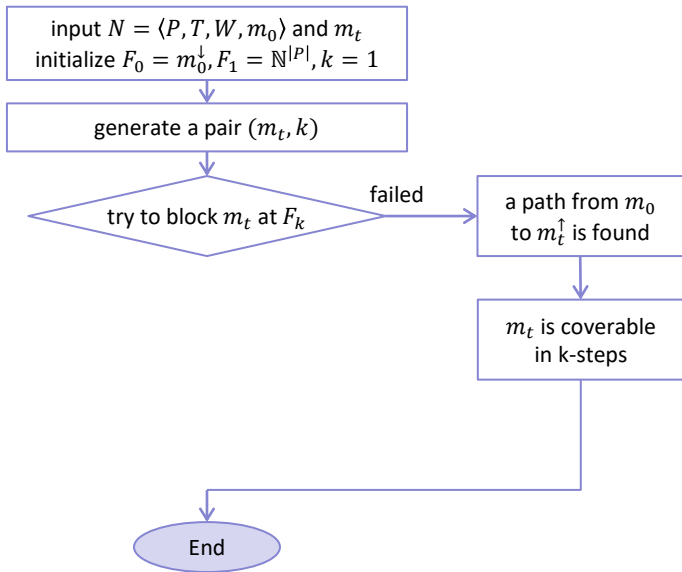
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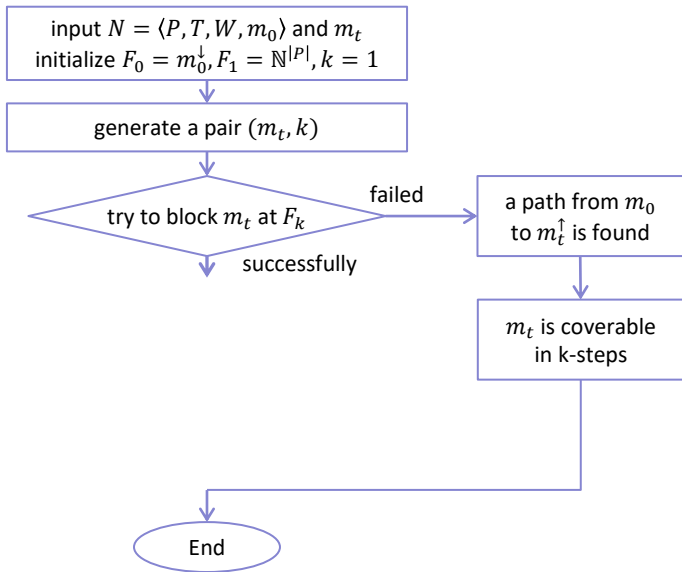
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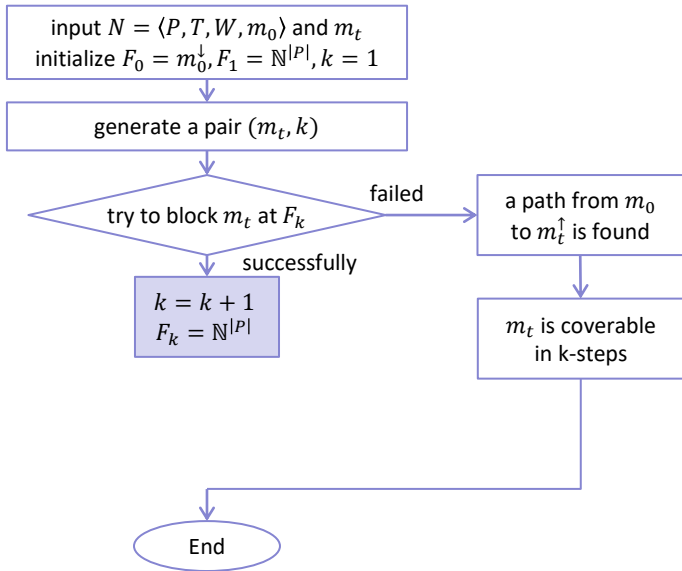
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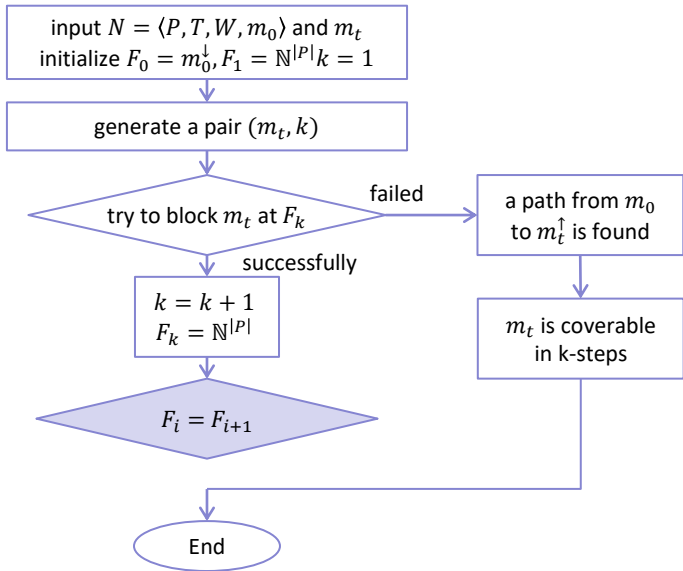
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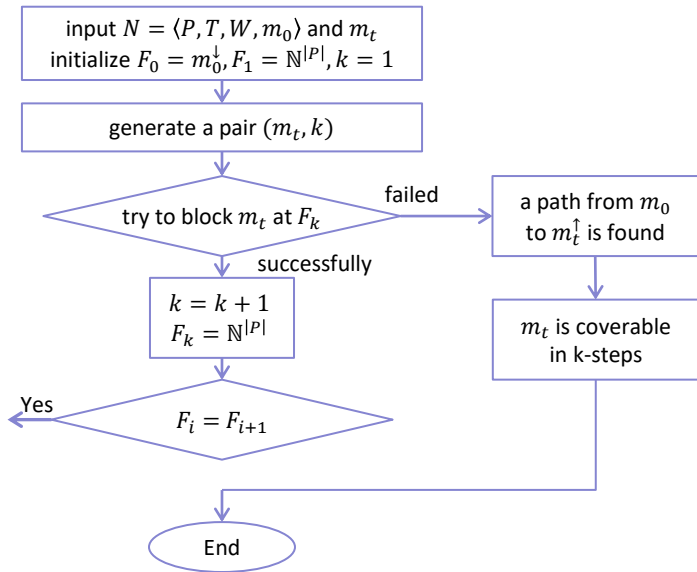
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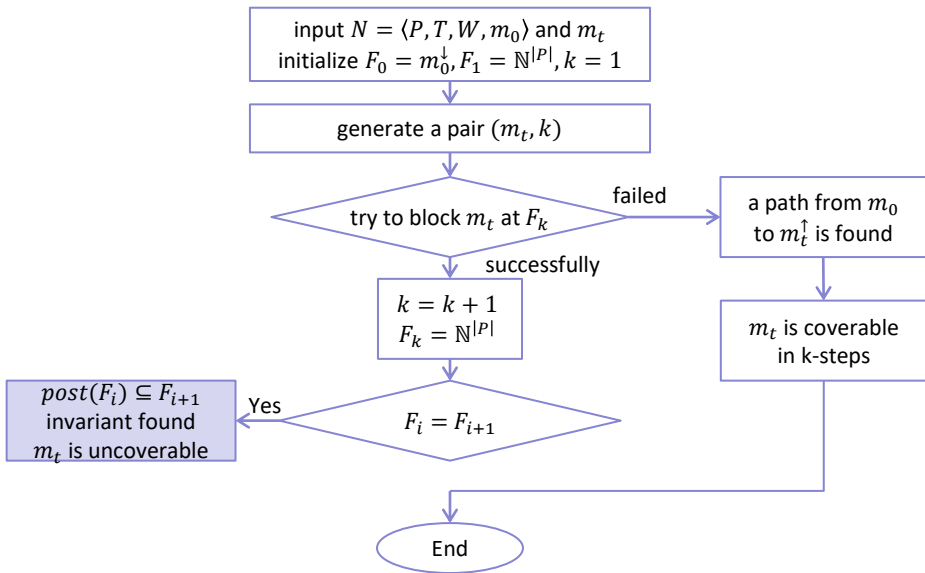
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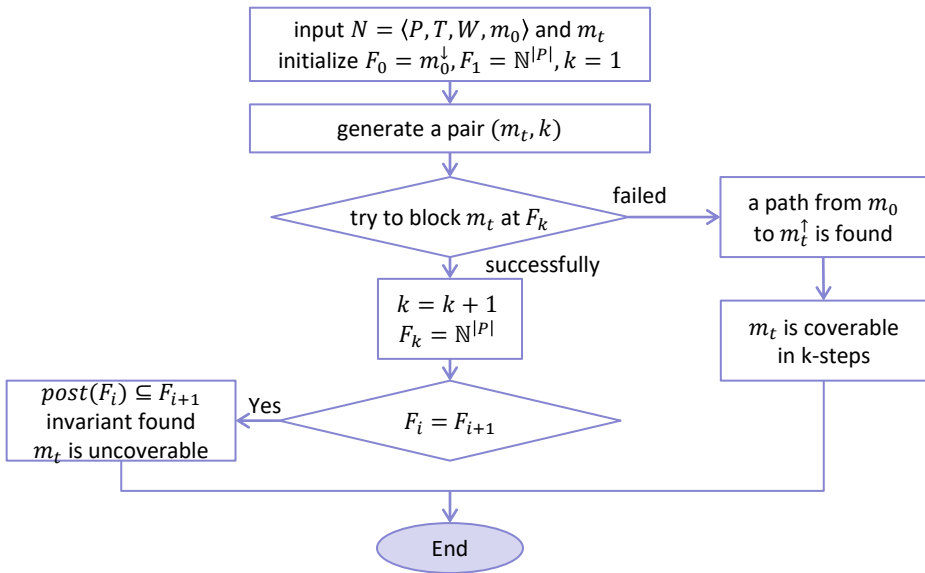
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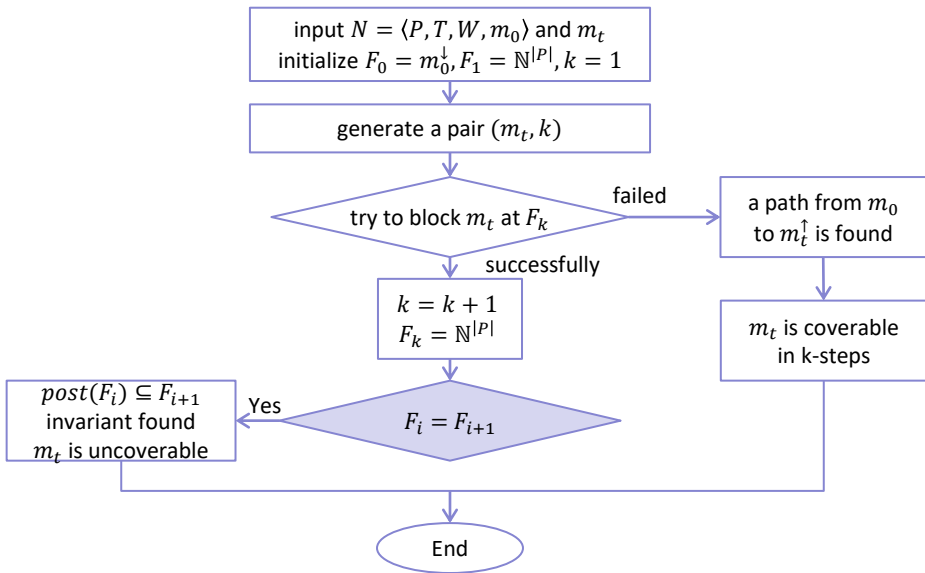
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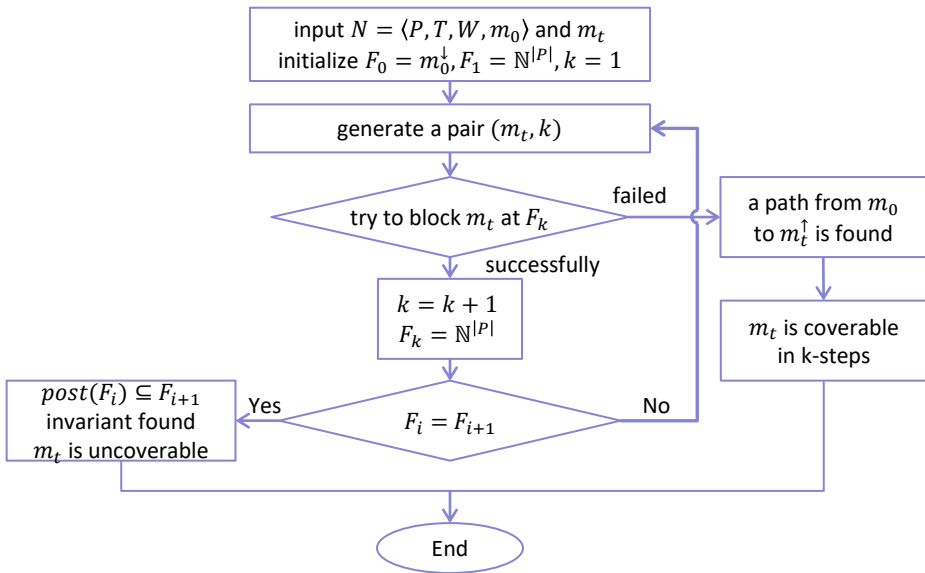
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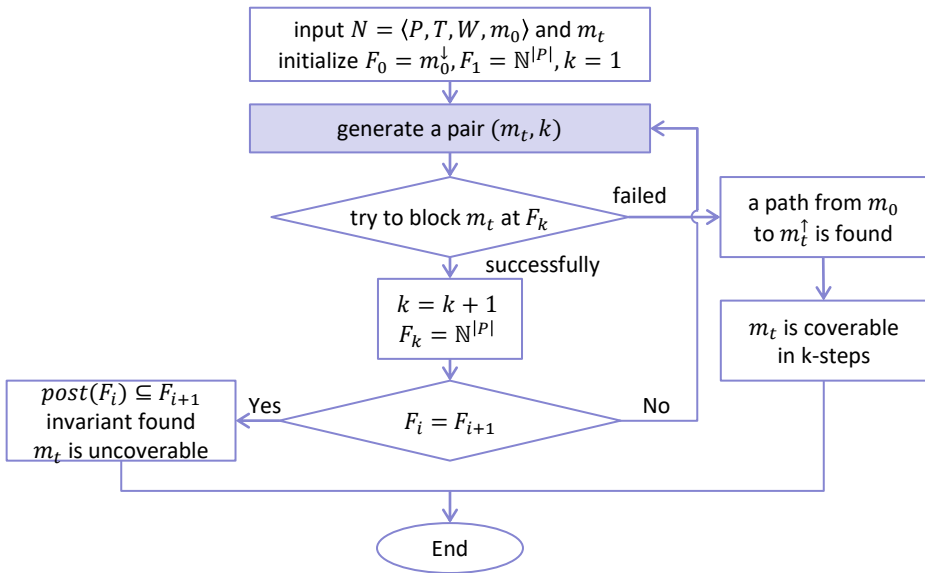
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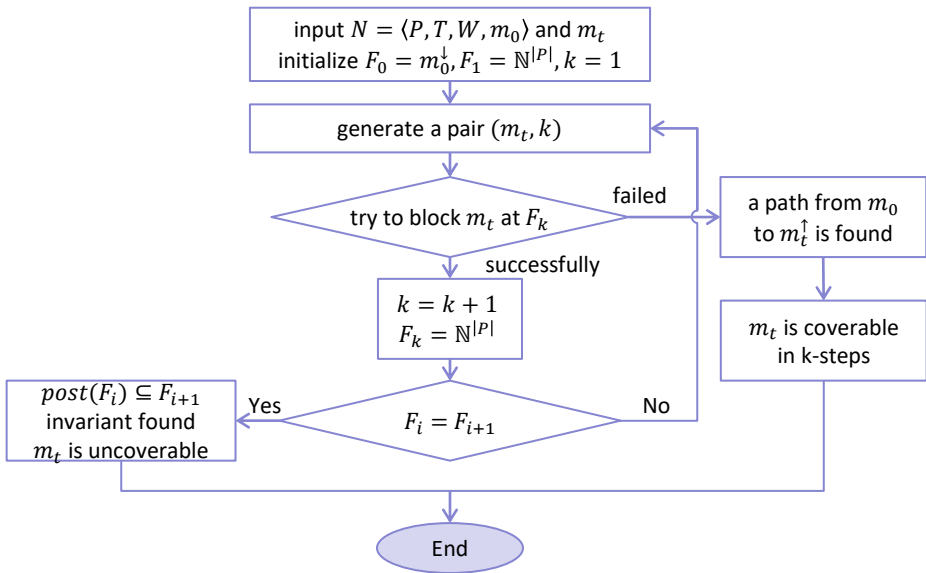
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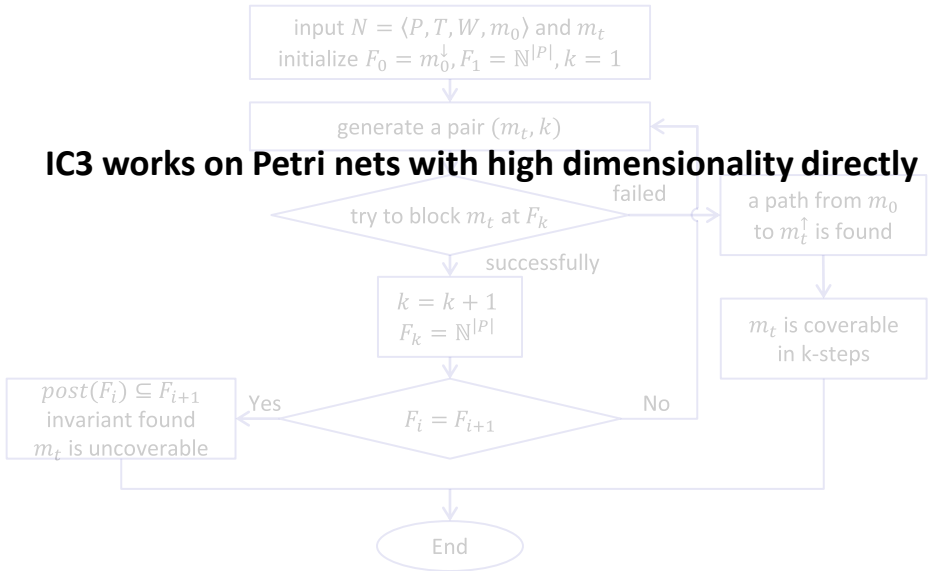


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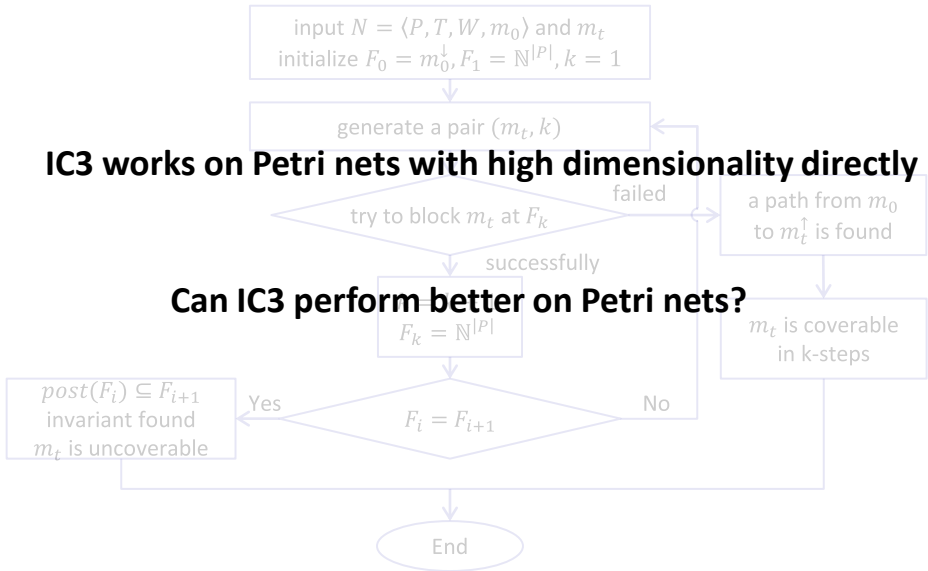


IC3 algorithm for Petri nets

IC3 works on Petri nets with high dimensionality directly



IC3 algorithm for Petri nets



Place-merge abstraction

Merge some places of original Petri net into **a single abstract place**, get an abstract Petri net with lower dimensionality.

Place-merge abstraction

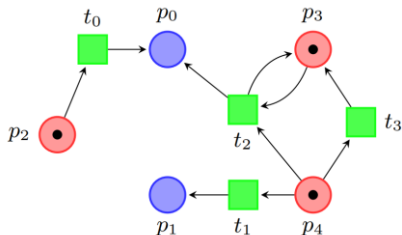
Merge some places of original Petri net into a **single abstract place**, get an abstract Petri net with lower dimensionality.

Definition

Given a Petri net $N = \langle P, T, W, m_0 \rangle$, where $P = \{p_1, p_1 \dots p_k\}$
- The abstraction function is a surjective function $\alpha: P \rightarrow \hat{P}$,
where $\hat{P} = \{\hat{p}_1, \hat{p}_2 \dots \hat{p}_{\hat{k}}\}$ and $\hat{k} \leq k$.

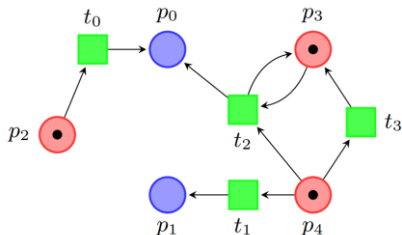
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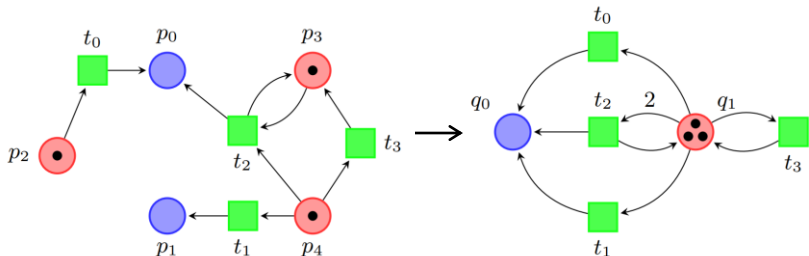
Merge some places of original Petri net into a **single abstract place**, get an abstract Petri net with lower dimensionality.



$$\alpha(p_0) = \alpha(p_1) = q_0$$
$$\alpha(p_2) = \alpha(p_3) = \alpha(p_4) = q_1$$

Place-merge abstraction

Merge some places of original Petri net into a **single abstract place**, get an abstract Petri net with lower dimensionality.



$$\alpha(p_0) = \alpha(p_1) = q_0$$
$$\alpha(p_2) = \alpha(p_3) = \alpha(p_4) = q_1$$

All weights of arcs are equal to 1 except for $W(q_1, t_2) = 2$.

Place-merge abstraction

Proposition

Given a Petri net $N = \langle P, T, W, m_0 \rangle$ and one of its abstractions $\hat{N} = \langle \hat{P}, T, \hat{W}, \hat{m}_0 \rangle$, m_t and its abstract version \hat{m}_t

- If m_t is coverable in N , then its abstract version \hat{m}_t is coverable in \hat{N} . But the converse does not hold.

Place-merge abstraction

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\hat{m}_t is uncoverable in \hat{N}

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\hat{m}_t is uncoverable in $\hat{N} \longrightarrow m_t$ is uncoverable in N

Place-merge abstraction

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\hat{m}_t is coverable in \hat{N}

Place-merge abstraction

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- If m_t is coverable in N , then its abstract version \hat{m}_t is coverable in \hat{N} . But the converse does not hold.

\hat{m}_t is uncoverable in \hat{N} \longrightarrow m_t is uncoverable in N

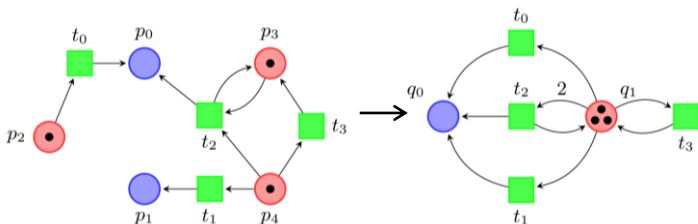
\hat{m}_t is coverable in \hat{N} $\not\Rightarrow$ m_t is coverable in N

Place-merge abstraction

Spurious counterexample

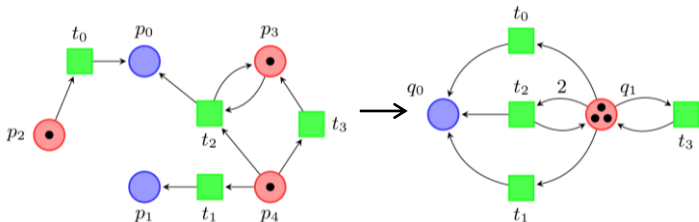
Place-merge abstraction

Spurious counterexample



Place-merge abstraction

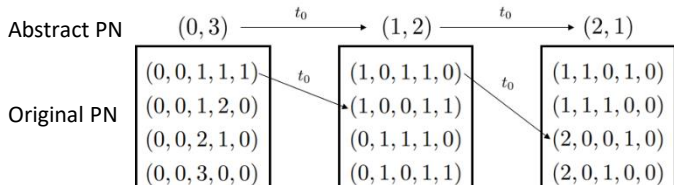
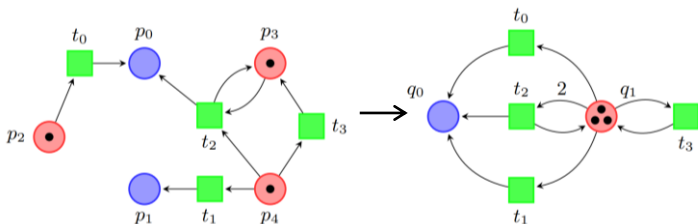
Spurious counterexample



Abstract PN $(0, 3) \xrightarrow{t_0} (1, 2) \xrightarrow{t_0} (2, 1)$

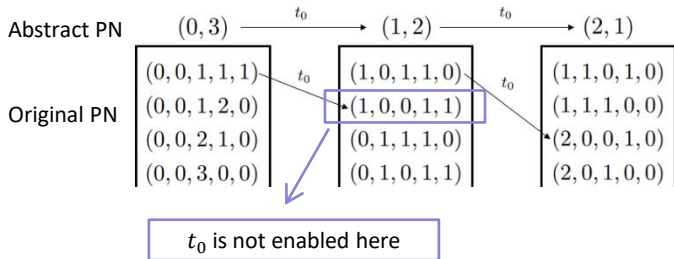
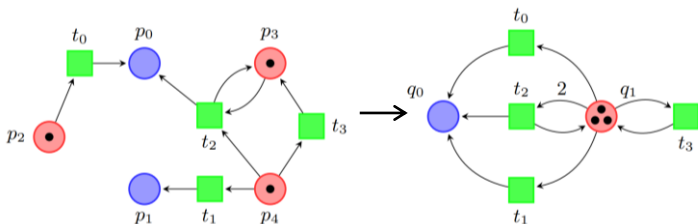
Place-merge abstraction

Spurious counterexample



Place-merge abstraction

Spurious counterexample



Place-merge abstraction

When a counterexample is spurious

Place-merge abstraction

When a counterexample is spurious

Counter-example $\pi = t_0 t_1 \dots t_{k-1}$ is not spurious iff

$$m_0 \xrightarrow{t_0} m_1 \xrightarrow{t_1} m_2 \xrightarrow{t_2} \dots \xrightarrow{t_{k-1}} m_k \wedge m_t \preceq m_k$$

Place-merge abstraction

When a counterexample is spurious

Counter-example $\pi = t_0 t_1 \dots t_{k-1}$ is not spurious iff

$$m_0 \xrightarrow{t_0} m_1 \xrightarrow{t_1} m_2 \xrightarrow{t_2} \dots \xrightarrow{t_{k-1}} m_k \wedge m_t \preceq m_k$$

The path π is spurious:

- ① t_i is not enabled at m_i ($0 \leq i < k$), **or**
- ② t_i is enabled at m_i ($0 \leq i < k$), but $m_t \not\preceq m_k$

Place-merge abstraction

How to refine an abstraction?

Place-merge abstraction

How to refine an abstraction?

t_i is not enabled at m_i ($0 \leq i < k$)

- extract places satisfying $m_i(p) < W(p, t_i)$
- merge these places into a new abstract place

Place-merge abstraction

How to refine an abstraction?

t_i is not enabled at m_i ($0 \leq i < k$)

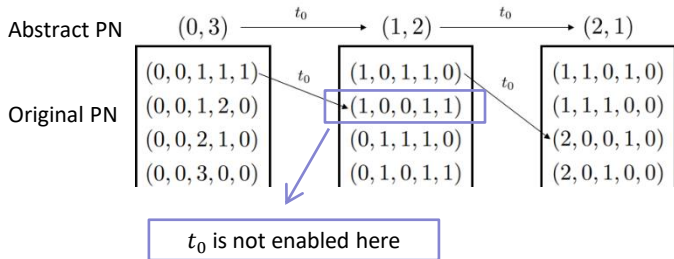
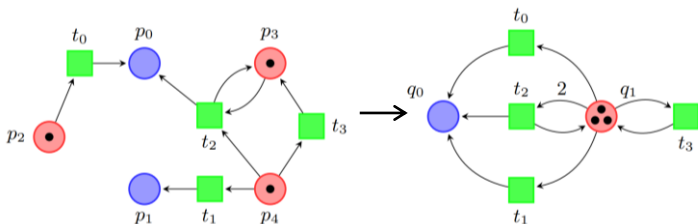
- extract places satisfying $m_i(p) < W(p, t_i)$
- merge these places into a new abstract place

t_i is enabled at m_i ($0 \leq i < k$), but $m_t \not\leq m_k$

- extract places satisfying $m_t(p) > m_k(p)$
- merge these places into a new abstract place

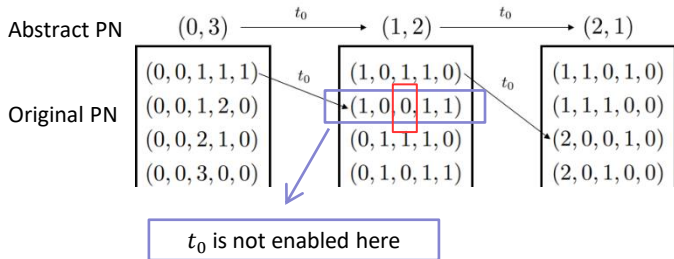
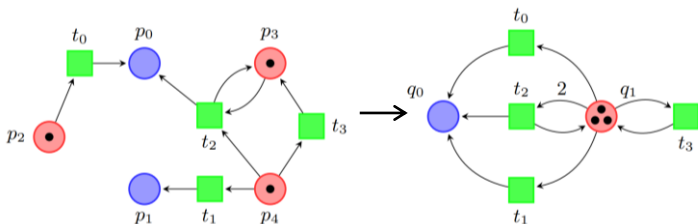
Place-merge abstraction

How to refine an abstraction?



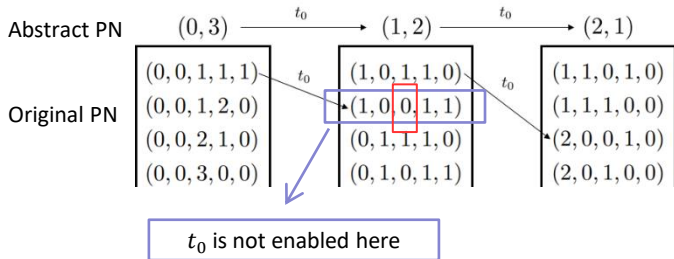
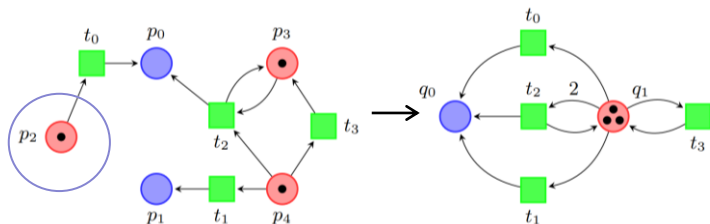
Place-merge abstraction

Abstraction refinement



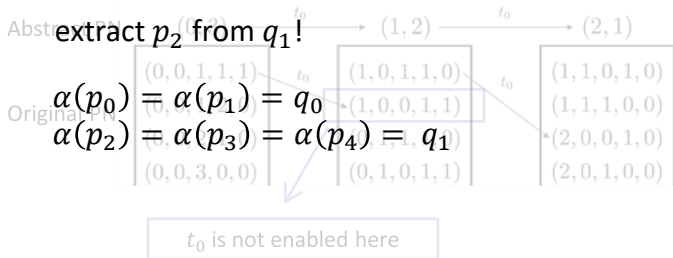
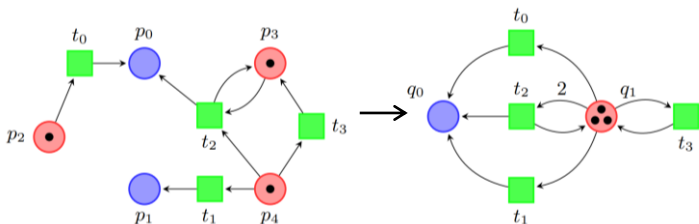
Place-merge abstraction

Abstraction refinement



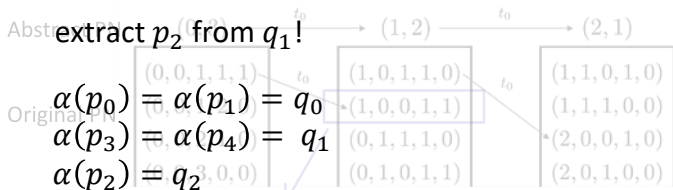
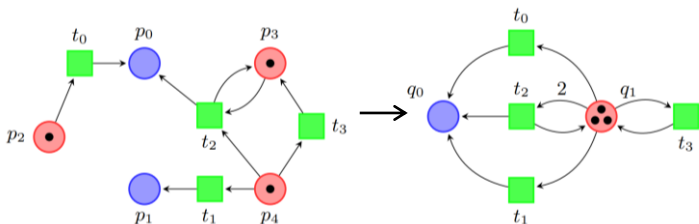
Place-merge abstraction

Abstraction refinement



Place-merge abstraction

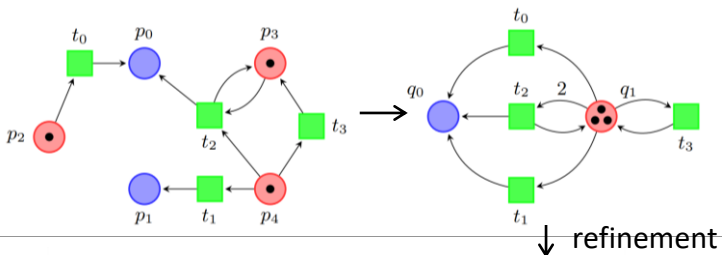
Abstraction refinement



t_0 is not enabled here

Place-merge abstraction

Abstraction refinement



extract p_2 from q_1 !

Abstract PN	$(0, 2)$	$(1, 2)$	$(2, 1)$
Original PN	$(0, 0, 1, 1, 1)$	$(1, 0, 1, 1, 0)$	$(1, 1, 0, 1, 0)$
	$(0, 0, 0, 1, 1)$	$(1, 0, 0, 1, 1)$	$(1, 1, 1, 0, 0)$
	$(0, 1, 1, 1, 0)$	$(0, 1, 1, 1, 0)$	$(2, 0, 0, 1, 0)$
	$(0, 1, 0, 1, 1)$	$(0, 1, 0, 1, 1)$	$(2, 0, 1, 0, 0)$

$$\alpha(p_0) = \alpha(p_1) = q_0$$

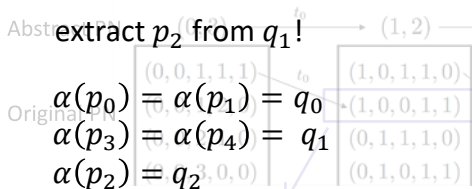
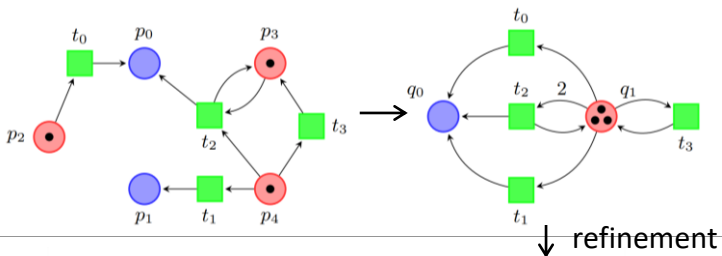
$$\alpha(p_3) = \alpha(p_4) = q_1$$

$$\alpha(p_2) = q_2$$

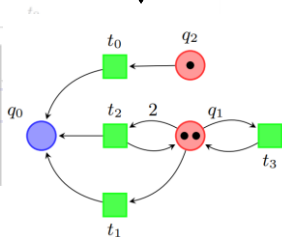
t_0 is not enabled here

Place-merge abstraction

Abstraction refinement



t_0 is not enabled here



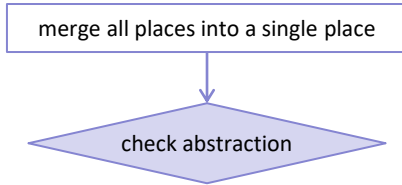
IC3+PMA algorithm

- Try to improve the outperformance of IC3
- IC3 is the core of IC3+PMA
- Place-merge abstraction reduces the dimensionality of PN
- IC3 works on the abstract PN with lower dimensionality

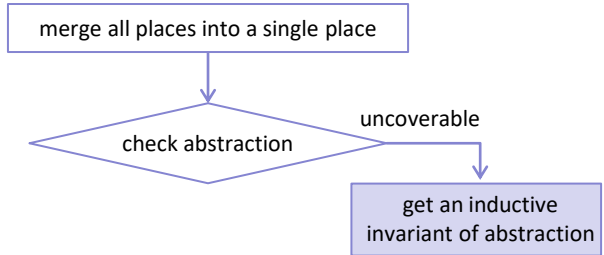
IC3+PMA algorithm

merge all places into a single place

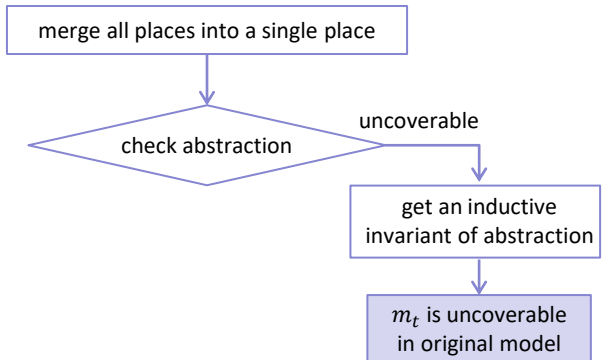
IC3+PMA algorithm



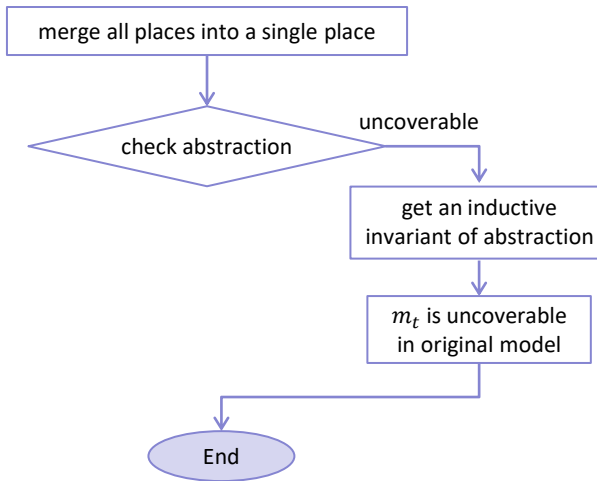
IC3+PMA algorithm



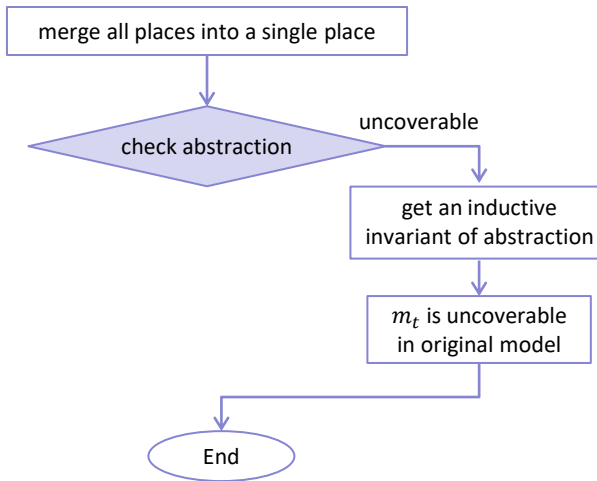
IC3+PMA algorithm



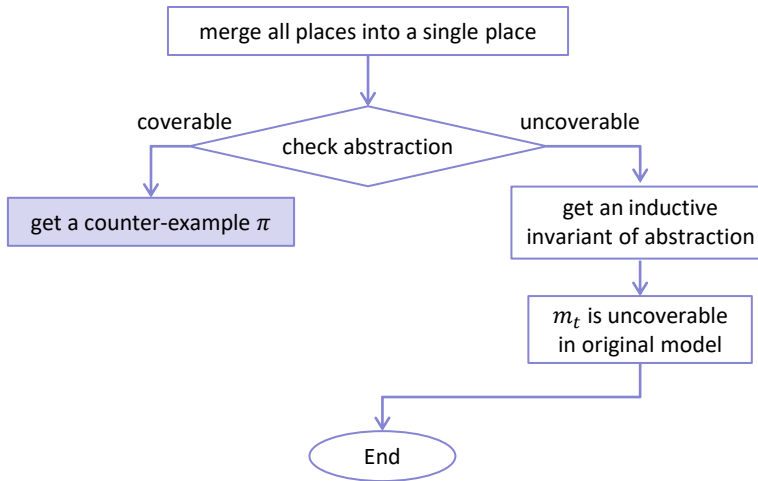
IC3+PMA algorithm



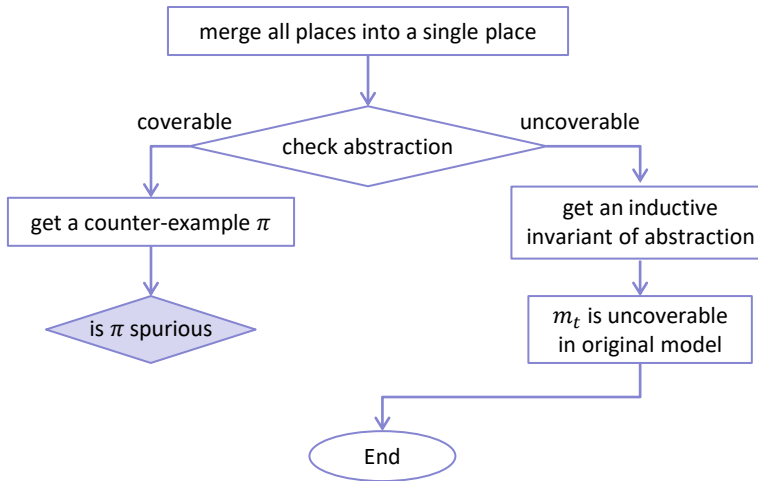
IC3+PMA algorithm



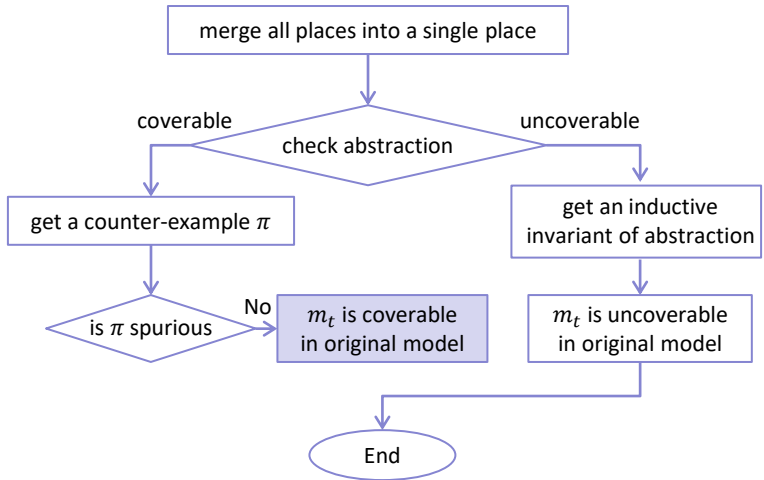
IC3+PMA algorithm



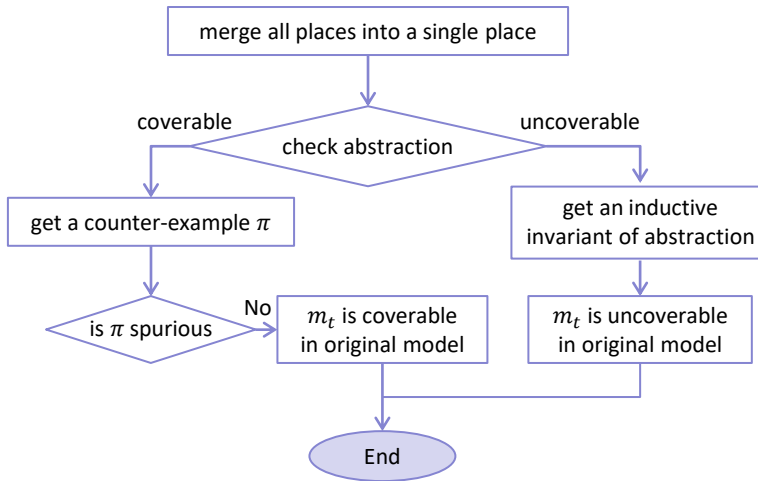
IC3+PMA algorithm



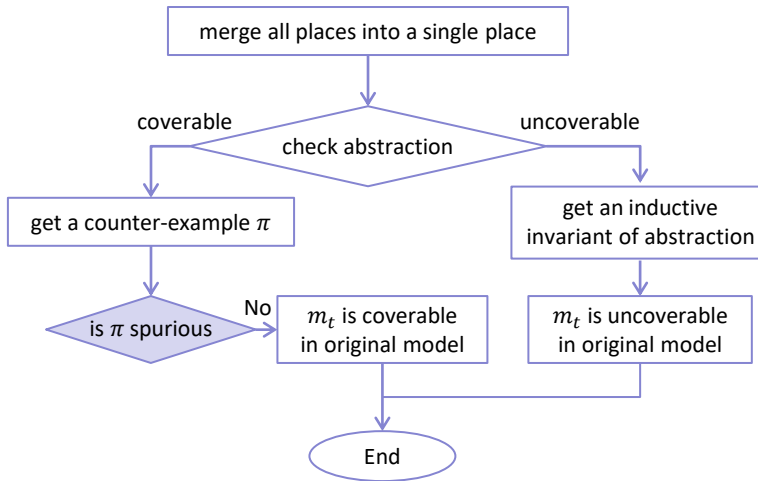
IC3+PMA algorithm



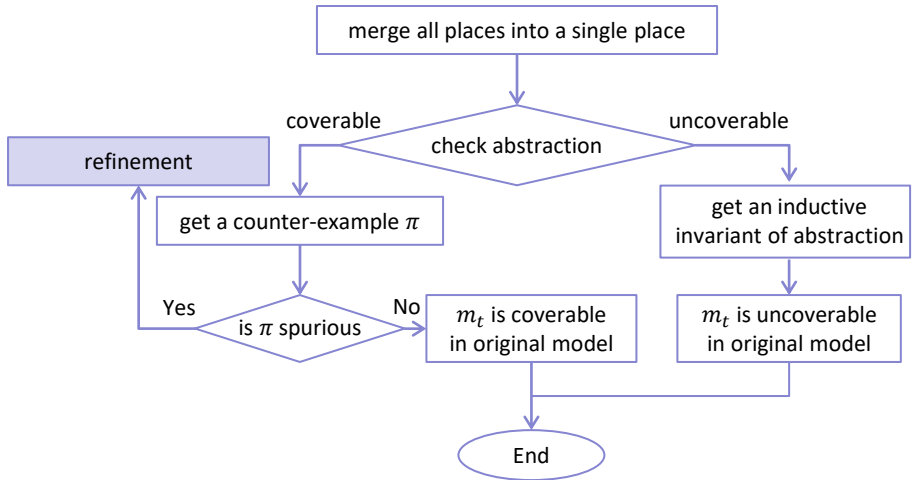
IC3+PMA algorithm



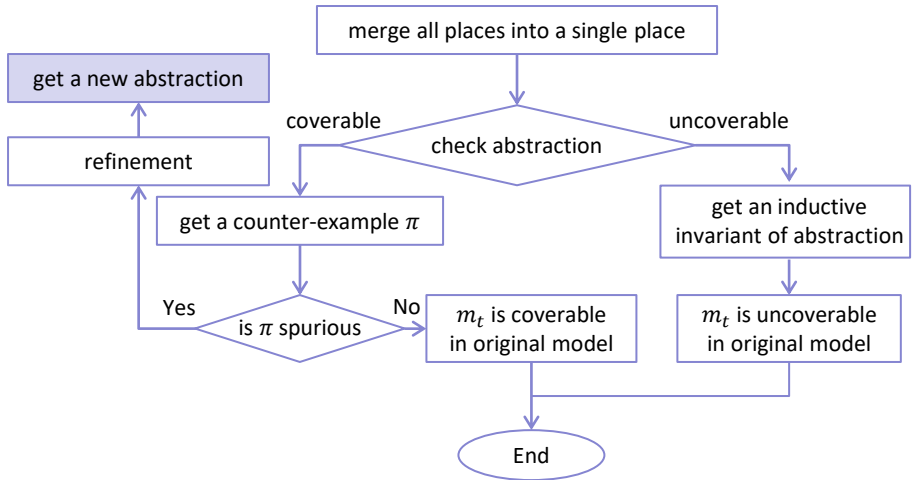
IC3+PMA algorithm



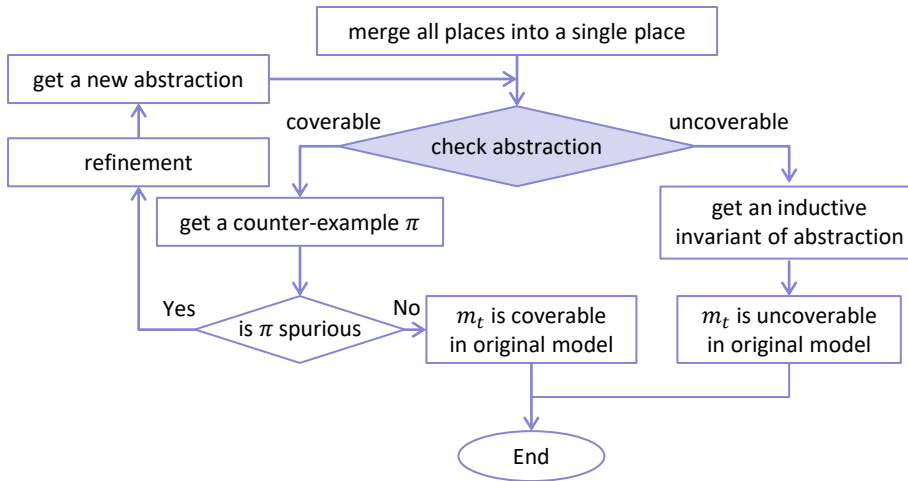
IC3+PMA algorithm



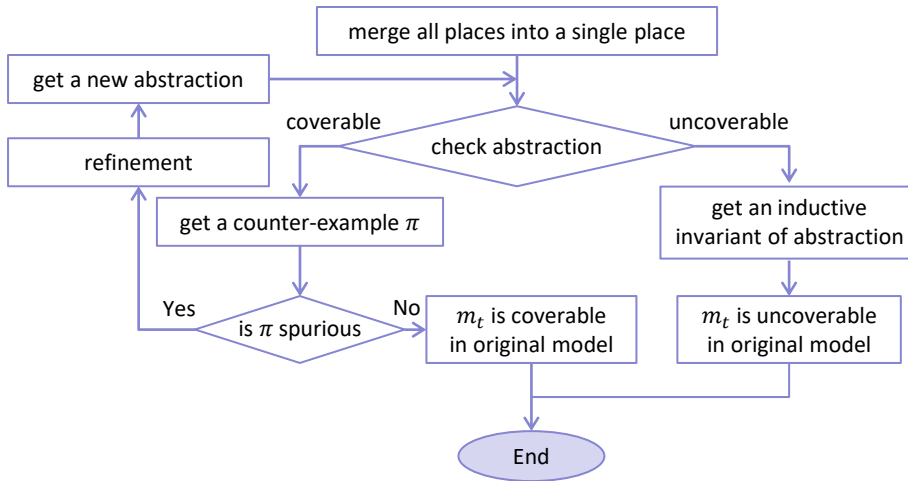
IC3+PMA algorithm



IC3+PMA algorithm



IC3+PMA algorithm



Experiments

- total 80 benchmarks
- compare running time between IC3 and IC3+PMA
- IC3+PMA outperforms IC3 on 53.75% of benchmarks
- dimensionality has decreased by 63.34% on average

Experiments

Benchmark	Places	IC3+PMA AbsPlaces	IC3+PMA Ref	IC3+PMA time(s)	IC3 time(s)
Uncoverable instances					
newrtsp	9	1	0	<0.01	0.06
kanban (bounded)	16	1	0	<0.01	1.22
manufacturing	13	1	0	<0.01	0.16
fms	22	4	3	<0.01	<0.01
fms_attic	22	4	3	0.01	0.04
mesh2x2	32	5	4	0.01	0.03
mesh3x2	52	5	4	0.02	0.08
pingpong	6	5	4	<0.01	<0.01
RandCAS 2	110	8	7	0.08	0.44
Conditionals 2	214	26	25	1.39	5.79
Coverable instances					
leabasicapproach	16	5	4	<0.01	<0.01
Dekker 1	41	27	25	2.08	3.23
DoubleLock1 1	64	35	32	11.26	13.31
Pthread5 1	80	47	44	97.28	Timeout
RandLock0 2	110	48	46	21.40	24.89
Spin2003 2	56	38	35	67.35	Timeout
Szymanski 1	61	46	44	19.62	32.69
Constants 1	26	14	13	0.03	0.03
FuncPtr3 1	40	16	13	0.19	0.33

IC3+PMA performs better

Experiments

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IC3+PMA performs better

Experiments

Benchmark	Places	IC3+PMA AbsPlaces	IC3+PMA Ref	IC3+PMA time(s)	IC3 time(s)
Uncoverable instances					
Peterson	14	10	8	0.35	0.13
Lamport	11	7	6	0.06	0.02
Ext. ReadWrite (small consts)	24	14	13	1.23	0.28
x0_AA_q1	312	#	#	Timeout	70.28
csm	14	9	8	0.19	0.02
Coverable instances					
RandCAS 1	48	34	33	0.85	0.67
StackCAS0 1	41	30	29	3.72	2.14
StackLock0 1	37	26	25	2.33	1.06
Lu-fig2 1	39	20	19	0.22	0.12
Lu-fig2 2	61	35	32	43.06	9.05

IC3+PMA performs worse

Experiments

Benchmark	Places	IC3+PMA AbsPlaces	IC3+PMA Ref	IC3+PMA time(s)	IC3 time(s)
Uncoverable instances					
Peterson	14	10	8	0.35	0.13
Lamport	11	7	6	0.06	0.02
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RandCAS 1	48	34	33	0.85	0.67
StackCAS0 1	41	30	29	3.72	2.14
StackLock0 1	37	26	25	2.33	1.06
Lu-fig2 1	39	20	19	0.22	0.12
Lu-fig2 2	61	35	32	43.06	9.05

- the efficiency of refinement method is not so high
- the way to deal with frames after refinement is not efficient

future work

- optimize the implementation to achieve better results
- apply the approach to analyze more properties and models

Thank You For Your Attention